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CARTAN ON COMPLEX PROJECTIVE GEOMETRY

Leçons de Géométrie Projective Complexe, par E. Cartan. Paris, Gauthier-Villars, 1931. vii+325 pp.

This volume has been prepared from the notes of M. F. Marty on a course of lectures given by Cartan at the Sorbonne during the winter of 1929–1930. As Cartan is one of the foremost geometricians of our time, one may, of course, expect that exposition and content of the treatise bear the marks of a master in his field. This is indeed the case, so that one may state without exaggeration that the book under review contains the most comprehensive and scientific treatment of complex geometry in existence.

It is hardly necessary to emphasize the importance of geometries in a complex domain. This is evident when we consider for example the transformation

$$z' = \frac{az + b}{cz + d}.$$

When a, b, c, d are real constants and z', z real variables, we are concerned simply with real projective geometry on a line, with its relatively simple content. But let the constants and variables involved be chosen from the complex domain, so that we are now concerned with the projective geometry of the complex line. Everybody realises how enormously the field has been enlarged and enriched. We need to think of the interpretation by circular transformations in the complex plane only, and its important applications in various mathematical fields, to realize the utility of extending geometric investigation into the complex domain.

The same relative importance attaches to the extension to higher complex spaces as carried out in a masterly fashion by Cartan. Assuming all quantities chosen from the complex domain, one may say that the content of projective geometry in the complex domain is formed by those properties of geometric forms or varieties which are invariant under the projective transformation or collineation (*homographie*)

$$\begin{aligned} x_1' &= a_{11}x_1 + \cdots + a_{1n+1}x_{n+1}, \\ x_2' &= a_{21}x_1 + \cdots + a_{2n+1}x_{n+1}, \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ x_{n+1}' &= a_{n+11}x_1 + \cdots + a_{n+1n+1}x_{n+1}, \end{aligned}$$

or the projective anti-collineation (*antihomographie*)

$$\begin{aligned} x_1' &= a_{11}\bar{x}_1 + \cdots + a_{1n+1}\bar{x}_{n+1}, \\ &\cdots\cdots\cdots\cdots\cdots \\ x_{n+1}' &= a_{n+11}\bar{x}_1 + \cdots + a_{n+1n+1}\bar{x}_{n+1}, \end{aligned}$$

in which $\bar{x}_i$ signifies the conjugate of x_i, and $|a_{ik}| \neq 0$.

The latter were introduced and studied by Juel and chiefly by Segre in 1889.

In his lectures Cartan presents the fundamental notions of these complex geometries and gives interpretations which connect them with Riemannian geometry. The work is divided into two parts. The first is given to the projective geometry of the complex line and its relation to hyperbolic (Lobatchewsky) geometry. The second is concerned with complex geometry in three dimensions. The last chapter deals with harmonic polynomials of complex projective space and their application to the representation of this space or of elliptic Hermitian space, by real algebraic varieties without singularities imbedded in a euclidean space of a convenient number of dimensions.

These few indications on the contents, which are altogether too extensive to give in detail, should convey an idea of the nature of Cartan's *Leçons*. Their reading does not require more than ordinary mathematical preparation, a knowledge of the cross ratio and a notion of Riemannian space.

To anyone who is interested in this field with a view to research, Cartan's book may be highly recommended as an indispensable background for further progress.

ARNOLD EMCH

NEW MATHEMATICAL TABLES

Mathematical Tables. Vol. 1. Prepared by the Committee for the Calculation of Mathematical Tables. London, British Association for the Advancement of Science, 1931. xxxv+72 pp.

Standard Four-Figure Mathematical Tables. By L. M. Milne-Thompson and L. J. Comrie. London, Macmillan and Company, 1931. xvi+245 pp.

The first activity of the British Association for the Advancement of Science in the preparation of mathematical tables appeared in a report of a committee published in 1873. "The purposes for which the Committee was appointed are twofold, viz. (1) to form as complete a catalogue as possible of existing mathematical tables, and (2) to reprint or calculate tables which are necessary for the progress of the mathematical sciences." This Committee, with changes in personnel, has been active for most of the time during the last sixty years. In the lists of members are to be found the names of Cayley, Stokes, Sir W. Thomson, Glaisher, Lord Rayleigh, Greenhill, Sylvester, Pearson, and many other well known British mathematicians. The only American to take part in the work as a committee member appears to have been A. G. Webster.

It was at first intended that tables should be published independent of the annual Report of the Association, and some tables have been so published, but most of the results have been included in the annual reports. The Association is now carrying out the original plan of separate publication, the first step of which is explained in the following quotation from the preface. "For several years the question of collecting into book form the tables from its reports has been before the Committee, but it was apparent from the first that the simple plan of reprinting existing material would produce a heterogeneous volume neither useful nor creditable. There were gaps in the ranges of the arguments of some of the functions, natural when the tabulation had been performed at

different times for special purposes, but intolerable if tables were to be issued for general use. In the case of the Bessel functions, the functions tabulated did not form in any sense a complete collection. Lastly, the original tables offered no facilities for interpolation. Two years ago the Committee decided that these difficulties must not impede publication indefinitely, and that, if the Bessel functions were reserved for an independent volume, definite progress could be made. Even with this limitation the labor of extending tables, computing differences, and checking has been heavy. When this work was completed, the tables were printed and proofs were read against the manuscript copy. The tables were then stereotyped and the proofs from the plates were checked rigorously by differencing or by mechanical integrating."

The preparation of this book, containing sixteen tables, was begun by R. A. Fisher and completed by J. Henderson. The introduction contains references to other similar tables. We give below a list of the tables in this volume, with a brief description of each.

I. *Multiples of* $\frac{1}{2}\pi$. This table gives the values to 15 decimal places of $n\pi/2$ for integral values of n from 1 to 100. Table I makes it possible to bring any angle within the range of Table III.

II. *Circular Functions.* This table gives the values to 15 decimal places of sines and cosines of angles measured in radians at intervals of 0.1 from 0.0 to 50.0. Table II, sections of which have appeared in the Reports from 1916 to 1928, was originally constructed because of its applications in the asymptotic expansion of transcendental functions.

III. *Circular Functions.* This table gives the values to 11 decimal places of sines and cosines of angles measured in radians at intervals of 0.001 from 0.000 to 1.600.

IV. *Hyperbolic Functions.* This table gives the values to 15 decimal places of $\sinh \pi x$ and $\cosh \pi x$ at intervals of 0.0001 from $x=0.0000$ to $x=0.0100$. Table IV is intended for use as an auxiliary table to Table V. It was calculated by L. J. Comrie, and is here published for the first time.

V. *Hyperbolic Functions.* This table gives the values to 15 decimal places of $\sinh \pi x$ and $\cosh \pi x$ at intervals of 0.01 from $x=0.00$ to $x=4.00$.

VI. *Hyperbolic Functions.* This table gives the values to 15 decimal places of $\sinh x$ and $\cosh x$ at intervals of 0.1 from 0.0 to 10.0.

VII. *Exponential Integral.* The exponential integral, which is of importance in connection with formulas for the number of primes less than x, is defined as

$$\mathrm{Ei}(x) = \int_{\infty}^{-x} \frac{e^{-u}}{u}\, du.$$

The first part of the table gives the values to 11 decimal places of $\mathrm{Ei}(x) - \log_e x$ and $\mathrm{Ei}(-x) - \log_e x$ at intervals of 0.1 from $x=0.0$ to $x=5.0$. The second part of the table gives the values to ten or eleven significant figures of $\mathrm{Ei}(x)$ and $-\mathrm{Ei}(-x)$ from $x=5.0$ to $x=15.0$.

VIII. *Sine and Cosine Integrals.* The sine integral $\mathrm{Si}(x)$ and the cosine integral $\mathrm{Ci}(x)$ are here defined as follows:

$$\mathrm{Si}(x) = \int_0^x \frac{\sin u}{u}\, du, \qquad \mathrm{Ci}(x) = -\int_x^{\infty} \frac{\cos u}{u}\, du.$$

These functions have applications in connection with the tabulation of derivatives of the Bessel functions and with various problems, especially in radio engineering. The first part of the table gives the values to 11 decimal places of $\mathrm{Si}(x)$ and of $\mathrm{Ci}(x)-\log_e x$ at intervals of 0.1 from $x=0.0$ to $x=5.0$. The second part gives the values to 10 decimal places of $\mathrm{Si}(x)$ and $\mathrm{Ci}(x)$ at intervals of 0.1 from $x=5.0$ to $x=20.0$, and the third part gives the values at intervals of 0.2 from $x=20.0$ to $x=40.0$.

IX. *Factorial Function.* This name is used in place of the more common Gamma Function, the notation being

$$x! = \Gamma(1+x) = \Pi(x).$$

The introduction explains some of the many useful formulas for the calculation of this function and the table gives the values to 12 decimal places of $x!$ at intervals of 0.01 from $x=0.00$ to $x=1.00$.

X. *Integral of Logarithmic Factorial Function.* The function here tabulated is

$$1+\int_0^x \log_{10} t!\,dt,$$

and the values to 12 decimal places are given at intervals of 0.01 from $x=0.00$ to $x=1.00$.

XI. *Digamma Function.* The function $\phi(x)$ is defined by

$$\phi(x) = \sum_{r=1}^{\infty} \frac{x}{r(r+x)}.$$

It is related to the logarithmic derivative of the factorial function by the formula

$$\frac{d}{dx}\log_e x! = \phi(x) - \gamma,$$

where γ is Euler's constant. Values of the logarithmic derivative are tabulated to 12 decimal places at intervals of 0.01 from $x=0.00$ to $x=1.00$ and at intervals of 0.1 from $x=10.0$ to $x=60.0$. Formulas are given in the introduction by which values of the function can be found for values of x between 1 and 10 and also for values of x beyond those given in the table.

XII. *Trigamma Function.* This is the derivative of the digamma function, that is, the trigamma function is

$$\frac{d^2}{dx^2}\log_e x! = \sum_{r=1}^{\infty} \frac{1}{(r+x)^2}.$$

This is a new table, calculated by A. Lodge. The intervals and ranges are the same as in Table XI.

XIII. *Tetragamma Function.* The name signifies the third logarithmic derivative of the factorial function, that is,

$$\frac{d^3}{dx^3}\log_e x! = -2\sum_{r=1}^{\infty} \frac{1}{(r+x)^3}.$$

The table is new and was calculated by A. Lodge. The intervals and ranges are the same as in Table XI.

XIV. *Pentagamma Function.* This is the last of the series of logarithmic derivatives of the factorial function to be given in this volume. The function is

$$\frac{d^4}{dx^4}\log_e x! = 6\sum_{r=1}^{\infty}\frac{1}{(r+x)^4}.$$

The table was calculated by A. Lodge and J. Wishart, and, as far as is known, is the only table of the function in existence. It extends over the same region as the three preceding tables, except that the first part (from $x=0.00$ to $x=1.00$) is given to only 10 decimal places.

XV. *Hh* Functions.

XVI. $\{Hh_0(x)Hh_2(x)\}/\{Hh_1(x)\}^2$.

The functions in the last two tables in the first volume find their applications in connection with the frequency functions of statistics. The *Hh* functions are defined by the relations

$$Hh_0(x) = \int_x^{\infty} e^{-(1/2)x^2}dx, \qquad Hh_n(x) = \int_x^{\infty} Hh_{n-1}(x)dx,$$

where n is a positive integer. Each function is thus the negative differential coefficient of the succeeding one, a property which may be used for purposes of interpolation. The method of interpolation is explained in a general introduction by J. O. Irwin.

For negative subscripts we have (Hermite functions)

$$Hh_{-1}(x) = -\frac{d}{dx}Hh_0(x) = e^{-(1/2)x},$$

$$Hh_{-n}(x) = -\frac{d^n}{dx^n}Hh_0(x) = -\frac{d^{n-1}}{dx^{n-1}}e^{-(1/2)x}.$$

The function in Table XVI is provided for use in solving the problem of fitting a truncated normal curve. An account of the properties and applications is given by R. A. Fisher.

The functions tabulated have been calculated by J. R. Airey. In Table XV the values of $Hh_n(x)$ are given to 10 decimal places for integral values of n from 0 to 21 and for different ranges of the argument. For negative values of n from -1 to -7, the number of decimal places varies from 0 to 10.

In the printing of these tables the mechanical work is excellent. They are in clear type on good quality of paper. The size of page (8 in. by 11 in.) allows an arrangement which is nowhere crowded and yet is not too large for convenient use.

The British Association is rendering most valuable service by its support of the laborious and expensive project of constructing mathematical tables, which are of such great importance in so many fields of application, and the individuals responsible for the work are to be congratulated on the results of their labors as shown in the first volume.

The Secretary to the British Association Mathematical Tables Committee is Dr. L. J. Comrie, one of the authors of *Standard Four-Figure Mathematical Tables.* In addition to those usually included in such a collection, this book

contains many new tables. Among these are an extended table of natural logarithms and of the trigonometric functions for angles in radians, and a table of inverse trigonometric and hyperbolic functions. A complete list follows. For those unfamiliar with the terminology it may be explained that Erf(x) (Table XVII) is a form of the probability integral.

I. Logarithms. II. Antilogarithms. III. Addition Logarithms. IV. Subtraction Logarithms. V. Roots, Powers and Reciprocals. VI. Some Functions of the Integers 1−100 (n^2, $\sqrt{n}$, $1/\sqrt{n}$, $1/n$, $1/n^2$, n^3, log $n!$, $0.6745/\sqrt{n}$, $0.6745/\sqrt{n(n-1)}$, $2\pi/n$, $1/n$). VII. Natural and Logarithmic Trigonometrical Functions (Degrees and Decimals). VIII. Natural and Logarithmic Trigonometrical Functions (Degrees and Minutes). IX. Natural and Logarithmic Trigonometrical Functions (Radians). X. Natural and Logarithmic Hyperbolic Functions. Powers of e. XI. Multiples of the Modulus M. XII. Inverse Trigonometrical and Hyperbolic Functions. Natural Logarithms. XIII. The Gudermannian in Radians. XIV. The Inverse Gudermannian. XV. Multiples of $1/M$. XVI. The Gamma Function. XVII. Erf(x). XVIII. Coefficients of the Second Difference. XIX. Conversion of Radians to Degrees and Degrees to Radians.

The last thirty pages of the book are devoted to a collection of the formulas of elementary mathematics, including the calculus and differential equations.

A unique feature of this work is that it is published in two editions. In Edition A the logarithms of numbers less than unity have been increased by 10. In Edition B the logarithms of numbers less than unity are printed with negative characteristics.

The size of the page is only slightly smaller than that of the British Association Tables, and the mechanical features show the same high quality. A convenient adjunct is a detached sheet of cardboard containing proportional parts of integers from 11 to 175.

W. R. LONGLEY

DICKSON ON THEORY OF NUMBERS

Studies in the Theory of Numbers. By L. E. Dickson. The University of Chicago Press, 1930. 10+230 pp.

More than a hundred years have elapsed since Gauss laid the foundation of the theory of ternary quadratic forms, developed by him only so far as was necessary for his immediate purposes. At the end of that section of *The Disquisitiones* dealing with ternary forms, Gauss points out for future investigators an immense field for research; namely, the arithmetical investigation of forms of higher degrees as well as of quadratic forms with more than two variables.

Despite the efforts of the greatest mathematicians up to our day, very little has been achieved in regard to forms of higher degrees, even in the binary case. On the contrary, the development of the theory of quadratic forms with three and more variables, in the hands of men like Eisenstein, Hermite, Smith, Minkowski, A. Meyer, and Voronoi, not to speak of other important contributors in this field, was crowned with considerable success. Their work is scattered in different publications and therefore is not easily accessible. Moreover, due to the natural complexity of the matter, it needs careful revision, since even the greatest among mathematicians are not proof against errors, which are sometimes slight and sometimes more serious, as the case may be.

The first attempt to present the theory of quadratic forms with three or more variables in its essential parts was made by Bachmann in two valuable volumes published in 1898 and 1923. Paying all due respect to an easy presentation of difficult matters and to the variety of subjects included in Bachmann's work, one cannot refrain from feeling that there still remains much to be desired. The main criticism of Bachmann's work, in the opinion of the reviewer, is that it lacks the critical revision of results and the proofs found in the original papers; so it often happens that defects for which even authoritative investigators should be held responsible are reproduced in Bachmann's book without change. One must be always on guard when trying to use Bachmann's *Arithmetik der Quadratischen Formen* as a reference book in his own research.

Under such circumstances, those who cultivate the field of quadratic forms —and their number, fortunately, is increasing—cannot but welcome the valuable volume under review. The rather indefinite title does not reveal that the book deals almost exclusively with questions pertaining to the arithmetical theory of ternary and quaternary quadratic forms. It is by no means a treatise on quadratic forms, but a presentation of some important selected topics of which Professor Dickson has made a profound study, and which he has enriched by valuable personal contributions. The reviewer and many others would certainly like to see a complete treatise on quadratic forms of the same high qualities as Professor Dickson's book, but it seems the time is not ripe for such an arduous enterprise, inasmuch as many important parts in the theory of quadratic forms are still in a state of infancy; for instance, the reduction theory of the indefinite (even ternary) forms.

Although limited in scope, Professor Dickson's book presents exactly what we miss in Bachmann's book: the carefully revised work of his predecessors,

with consequent perfect reliability; many original results; and, finally, the concise and yet clear exposition so characteristic of the author.

The book is divided into three loosely connected parts. The first begins with the explanation of the fundamental properties of general quadratic forms, and is followed by an excellent introduction to the theory of ternary forms. Incidentally, in the second chapter Professor Dickson proves that an indefinite ternary quadratic form representing all integers (he calls it a *universal form*) must represent 0 for integral values of variables not vanishing simultaneously. The main object of the first part, however, is a thorough revision and completion of an extensive theory developed by Arnold Meyer in several papers which are by no means easy to read. Eisenstein once made a rather vague remark that, for indefinite ternary quadratic forms with relatively prime odd invariants, each genus contains one class. Meyer tried to prove this remark, and in general to find the number of classes contained in each genus under more general conditions. In the course of a rather complicated investigation, Meyer, it seems, did not escape making erroneous statements, and we must surely feel grateful to Professor Dickson for putting this important theory on a sound basis. In the final chapter of the first part Professor Dickson applies the previously obtained results to the proof of an interesting statement, also due to Meyer; namely, that an arbitrary indefinite form with more than four variables is a zero form. The proof developed here is a first complete proof of this theorem on the lines outlined by Meyer. A closely related question concerning the conditions of solvability of the diophantine equation $ax^2+by^2+cz^2+dt^2=0$ is also treated in the same chapter. One regrets that no mention is made of a recent work by H. Hasse, which seems to offer a more genuine approach to questions of this kind.

The second part of the book especially appeals to the sentimental feeling of the reviewer, since it deals with questions studied by the late Professor A. A. Markoff, whom he has the right to revere as his teacher. Korkine, who, together with Zolotareff, published extremely beautiful investigations on precise limits of minima of definite forms, suggested to Markoff that he consider similar problems for indefinite quadratic forms. Markoff, who then was only twenty-two years of age, in a short time attained most startling results in the case of binary forms, and published them first in the Mathematische Annalen, and later in more developed form as a thesis presented to the physico-mathematical section of the University of St. Petersburg in 1880. Many years later Markoff extended his work to ternary and quaternary forms, but here the results obtained by him are not as complete as in the case of binary forms.

Such are the questions dealt with in the second part of the *Studies*. Professor Dickson gives an elaborate exposition of Markoff's results in the binary case and rectifies and completes his investigations in the ternary case. The author then proceeds to explain new results obtained by Dr. A. Oppenheim for quaternary forms. It seems, however, that neither Professor Dickson nor Dr. Oppenheim was aware of a Russian paper by Markoff in the Bulletin de l'Académie Impérial des Sciences de St. Petersbourg, 1902, where the following result can be found: For indefinite quaternary forms of determinant D the precise limit of the minimum is $(4|D|/7)^{1/4}$ and this limit is attained for the forms equivalent to $\phi_0=(4|D|/7)^{1/4}[(x-\frac{1}{2}t)^2+(y-\frac{1}{2}t)^2+(z-\frac{1}{2}t)^2-7/4t^2]$.

The precise minimum for the remaining forms is $(4|D|/9)^{1/4}$ and is attained in the class of forms equivalent to

$$\phi_1=(4|D|/9)^{1/4}(x^2+xy+y^2-2z^2-2zt-2t^2).$$

Any form not equivalent to ϕ_0 or ϕ_1 has a minimum less than $(4|D|/9)^{1/4}$.

The concluding chapter of the second part of *Studies* deals with tabulation of indefinite ternary quadratic forms not representing 0 and contains a table of non-equivalent forms with determinants not greater than 80 (with the exception of the determinant 68 where the equivalence of two forms $x^2-3y^2-2yz-23z^2$ and $x^2-7y^2-6yz-11z^2$ could not be decided).

A similar table extended to all determinants not greater than 50 was published earlier by Markoff with an important additional feature; namely, it includes for each form a set of linear homogeneous inequalities which define a domain surely containing only a limited number of representations of any integer by the corresponding form. It seems to the reviewer that the general proof that linear inequalities of such nature exist for any indefinite ternary form (not zero form) would be of the utmost importance and well worth investigation.

Passing to the third and last part of the *Studies*, which is devoted to miscellaneous investigations, we first call attention to Chapter 12, where the question of universal forms is resumed and brought to a solution in the case of ternary and quaternary forms.

The preceding chapter gives a very detailed elaboration of the reduction theory of positive ternary forms, in general based on Dirichlet's geometrical method, with some modifications intended to define reduced forms essentially in Eisenstein's manner. This is followed by a table of reduced positive ternary quadratic forms with determinants not exceeding 50. While it is very pleasant to have conditions whereby the reduced forms are uniquely defined, it seems that the approach to this problem by characterizing the unique reduced parallelipipedon is not the best. It is more natural to define a fundamental domain *in the space of the coefficients* of the form, as did Minkowski and, after him, Voronoi, in their profound investigations of the reduction theory of general positive quadratic forms. Applied to the ternary case, these general considerations lead easily to a perfect and elegant geometric solution of the reduction problem.

At the very end of Chapter 11, Professor Dickson gives the result due to Korkine and Zolotareff concerning the precise limit of minima of positive quaternary forms. The proof given here is unfortunately applicable only to quaternary forms, and we regret that no mention is made of the important later papers by Korkine and Zolotareff, whose work was continued first by Voronoi, and in recent times with great success by Professor H. F. Blichfeldt.

The last chapter of the book has little to do with the arithmetical theory of quadratic forms except in a remote sense. This chapter, prepared by Dr. Oppenheim, presents an elegant exposition of G. H. Hardy's analytical proof for the known formulas which express the number of representations of integers by sums of not more than eight squares.

In conclusion, we repeat that *Studies in the Theory of Numbers* is certainly a highly important contribution to the literature on quadratic forms, and that it should be consulted by everyone interested in this fascinating field of research.

J. V. USPENSKY

SHORTER NOTICES

Fundamentals of Electricity and Magnetism. By Leonard B. Loeb. New York, John Wiley and Sons, 1931. xi+432 pp. Price $4.00.

This is not intended to be a text-book for an intermediate course in electricity and magnetism as the title might suggest. It was written to satisfy the needs of engineering students in the University of California, who spread the study of general physics over two years. The book covers about the same ground as the chapters on electricity and magnetism of most text-books on college physics. Certain topics, however, are more fully discussed, e.g., circuits containing capacity, self-induction and resistance, Kirchoff's laws of divided circuits, and thermionics.

The book is the outcome of several years of demonstration lectures in the University of California and is written in lecture style. New concepts are usually introduced through experiments and formal definitions are given only after the concepts are fully explained. This makes the presentation informal and readable. An interesting feature of the book is the first chapter where an excellent account of the history of the general development of physical science is given.

H. M. DADOURIAN

Cours de Mécanique. By Paul Painlevé. Tome I. Paris, Gauthier-Villars, 1930. vi+664 pp.

This is the first volume of a series which is to present a course of lectures delivered at l'Ecole Polytechnique. The volume is divided into four books, the first of which is devoted to vector addition and multiplication. The second book is on the fundamental axioms of Newtonian mechanics, the most important of which may be stated as follows:

It is possible to adopt, once for all and for the entire universe, a measure of distance, a measure of time, and a system of reference axes such that the following principles always hold good: I. The constancy of the velocity of an isolated particle; II. The equality of action and reaction; III. The determinate character of the mutual accelerations of two isolated particles when their velocities and their distance apart at any instant are given; IV. The geometric addition of forces.

A reference system relative to which the foregoing four propositions are true is called an *absolute* system of axes; velocities and accelerations referred to such a system are called *absolute* velocities and accelerations. "The fundamental postulate of mechanics," says the author, "consists, therefore, in the admission of a system of absolute axes." Every system which has a uniform motion of translation relative to an absolute system is also an absolute system.

The third book is entitled *The general theorems of the dynamics of systems*, and deals mainly with the motion of a particle.

The fourth book is on the general theory of the equilibrium and motion of systems. It is less elementary than the third book. D'Alembert's principle, Lagrange's equations, and the principle of virtual velocities are here applied to problems of equilibrium and of motion.

The presentation is clear, logical, and rigorous as would be expected from a French scientist of the standing of Painlevé. The book deserves a prominent place on the reference shelf of every serious student of mechanics.

The only serious criticism I would offer could be made against almost all books on mechanics. It seems to me that the presentation of Newtonian mechanics lends itself to a degree of unity and integration comparable to that of the theory of relativity. The entire subject could be based upon a single fundamental principle from which all other principles, laws, and theorems could be derived as was done by Lagrange in his *Mécanique Analytique*. In such a presentation the geometric addition of forces, for example, would not be elevated to the status of a fundamental principle any more than the geometric addition of other vector magnitudes, such as velocities and accelerations.

H. M. DADOURIAN

Balistique Intérieure Théorique et Tables Numériques. By G. Sugot. Paris, Gauthier–Villars, 1928. xxx+130 pp.

This book falls into two parts which may be characterized respectively as formal and experimental. That is to say, the first part develops a theory in terms of certain parameters and functions which may depend upon the gun or the ammunition or both. The second part deals with the methods for determining these parameters or functions from the measurable quantities obtainable either in routine firings or in special experimental firings. It is not "experimental" in the sense of describing apparatus.

The first part begins with a chapter on the two fundamental equations of interior ballistics, namely, the energy equation and the expression of the law that the linear rate of burning of smokeless powder is proportional to a fixed power of the pressure. It is somewhat remarkable that practically all treatments of interior ballistics accept this form of the law of burning, although there is such wide diversity in the power selected, the exponent varying from unity down to one-half, or lower. At this stage Sugot makes no selection of exponent. His second chapter is a short one on ballistic similitudes. The third deals with the special case of powders of constant surface, that is, powders of such a grain form that the burning surface remains constant as the size decreases. The single perforated cylinder used in rifle powder approximates to this requirement. The fourth chapter deals with other grain forms, and is in this respect a generalization; but it is specialized in another direction by the assumption that the exponent in the law of burning is equal to unity, and by certain minor assumptions not made before. These are selected in such a way as to reduce the problem to the solution of a first-order linear differential equation. The third and fourth chapters consist largely in the discussion of the curves representing pressure, energy, fraction of powder burned, etc. as functions of projectile-travel, or a related variable, particularly as these curves are affected by changes in the conditions of loading or the design of the gun.

The subject matter of the second part, called by the author "Outillages Balistiques," may be described as dealing with the best ways of making bricks without straw. That is to say, the phenomena which actually take place inside a gun when it is fired are of such complexity that the meager data obtainable from measurable quantities leave a great part of the theory indeterminate. Thus

routine firings give muzzle velocity with some accuracy and maximum pressure with somewhat less. Special experimental work may give a time record of pressure in closed chamber firings, or of recoil in gun firings. (This book makes no mention of time records of pressure or projectile travel in gun firings.) Out of this rather scanty information one must answer not only the purely ballistic questions such as concern the law of burning, combustion temperature, composition of the powder gases, band resistance, but also various general questions of physics and chemistry, on which there are no experimental data obtainable elsewhere because of the combination of high temperature and high pressure involved. Chief among these are the characteristic equations and specific heats of the powder gases. To deal with these difficulties, two courses of procedure are open, leading on the one hand to the "outillage expérimental," on the other to the "outillage théorique." In the former, methods are selected mainly with a view to obtaining empirically the observed relations connecting conditions of loading with muzzle velocity and maximum pressure. In the latter a greater endeavor is made to formulate hypotheses which plausibly represent the actual phenomena. The difference is in a sense one of degree rather than kind. In the first method the selection of variables for empirical comparison must have some physical basis; in the second there must always be left various parameters or functions to be determined empirically. The author deals with both methods and various types of each, but devotes most attention to the "outillage théorique." The large number of numerical tables pertain to this part of the book.

In spite of the general excellence of the work it seems open to criticism for provincialism. The author makes only the most cursory reference to writers outside of France, and none to any outside the Latin countries, either in ballistics or relevant parts of general science. While it is undoubtedly true that interior ballistics has had by far the greatest development in France, nevertheless it seems unfortunate to ignore completely, for example, the German work on the conditions of chemical equilibrium in the powder gases. However, it may be said that the author omits no well-established phenomenon of major importance, and presents the principal mathematical methods that are useful.

L. S. DEDERICK

Introduction to Vector Analysis. By L. R. Shorter. London, Macmillan, 1931. xiv+355 pp.

In his preface the author says: "My object in writing this introductory book is to furnish a working knowledge of the subject, and so enable a mathematical student to continue his studies with ease in the many excellent existing textbooks. If, on the other hand, the reader is a student of Physics, this book should supply him with a sufficient working knowledge of the methods and enable him to apply them in his physical studies."

To the reviewer it seems that the book will meet both of these aims better than most books with similar purpose. The introduction to new ideas is clear and convincing though not burdened with oppressive rigor. Numerous examples worked out in full detail illustrate the theory, and it should be possible for a student to obtain from the text a working knowledge of the subject with little or no help from a teacher.

Chapters 1, 3, and 5 are entitled respectively *Addition*, *Multiplication*, and

Differentiation. Chapters 2, 4, and 6 consist entirely of illustrative examples worked out in complete detail, 138 such examples in all. While their value cannot be questioned, most American teachers would probably prefer to have some exercises left for solution by the student. The seventh and last chapter is entitled *Applications.* It seems to have absorbed the chapter which we might expect on integration. In it we find the theorems of Gauss and Stokes, and the usual applications of the differential and integral calculus of vectors to theoretical mechanics. One wonders why the author did not (in accordance with the plan adopted in the earlier chapters) follow Chapter 7 by a chapter of examples illustrating the theory. Such a chapter would have been most valuable.

The appearance of the book is up to the high standard maintained by its publishers. Fortunately the reader will not be greatly troubled by the following omissions and misprints which have been noted by Professor Currie, a colleague of the reviewer.

On page 115, line 11, primes have been dropped from y' and z'.

On page 144, in the formula for S_1^2, a minus sign should obviously be replaced by a plus.

On page 146, in Example 48, one should read "the (sum of the) squares." The words in parentheses are omitted.

On page 229, in line 6, the symbols $/\partial x$ have been dropped in two places.

On page 230, in line 4, a plus sign should be replaced by a minus.

On page 233, in line 4, the symbol $\times$ should be replaced by $+$, and the same error occurs in line 6, page 249.

On page 234, near the bottom of the page, a symbol has been dropped, and the sense destroyed.

L. L. DINES

Darstellende Geometrie. By Robert Haussner and Wolfgang Haack. III, *Zylinder, Kegel, Kugel, Rotations und Schraubenflächen, Schattenkonstruktionen, Axonometrie,* Berlin and Leipzig, Walter de Gruyter, 1931. 141 pp., 65 figures. M. 1.80.

This is volume 144 of the Sammlung Göschen and contains an elementary treatment of quadric and other simple surfaces, their intersection and development, etc., by descriptive geometric methods. The contents are those of the customary elementary text on descriptive geometry and are presented in a clear and pleasing manner.

One important feature, often neglected in elementary descriptive geometry texts, is the systematic application of affine relations between horizontal and vertical projection of a plane figure, of projection and rabattement, etc. There are many teachers of descriptive geometry who do not realize the enormous advantage of this method for constructive purposes. Moreover it opens the way to an intelligent understanding of important geometric relations, especially the idea of transformation.

From this standpoint the little book may be recommended to teachers and students alike.

ARNOLD EMCH

Pour l'Histoire de la Science Hellène. De Thalès à Empédocle. By Paul Tannery. Deuxième Édition par A. Diès, avec une préface de M. Federigo Enriques. Paris, Gauthier-Villars, 1930. xxiv+435 pp.

During the half century from about 1880 our knowledge of Greek mathematics has been largely revolutionized by the work of two men, Sir Thomas L. Heath in England and Paul Tannery in France. The work of both men represents the highest type of modern scholarship; it is interesting that both worked largely independently of any university connection.

Paul Tannery's contributions consisted essentially of a series of several hundred monographs throwing light upon the life and achievement of Greek philosophers, astronomers, and mathematicians. In this work Tannery had the devoted assistance of his wife through whose efforts his collected works and correspondence have now been assembled in twelve volumes. In the preface to this volume as it appeared in 1887 Paul Tannery dedicates the work to his wife, the indefatigable companion in his activities.

While these apparently scattered essays may seem to have had no great underlying unity, yet the volumes issued by Tannery on Greek Astronomy (*Pour l'Histoire de l'Astronomie Ancienne*, 1893), Greek Geometry (*Géométrie Grecque*, 1887), and this volume under review on Greek Philosophy, represent in these fields a new point of view, revolutionizing ancient views held and widely taught up to modern times.

The text of this work is left as it appeared in the edition of 1887. However certain tentative notes made by Paul Tannery from time to time in his own personal copy of the edition of 1887 have been included as footnotes. At the same time the references have been given to Diels' *Die Fragmente der Vorsokratiker*, which appeared first in 1903 and which has enjoyed three subsequent editions. A bibliography of pertinent works which have appeared since 1887 constitutes a very useful addition.

In this philosophical work the sections which interest the student of mathematics are particularly those which treat of the relation to oriental science, to Thales, and those on the Pythagorean arithmetic. The amazing material which has recently been discovered relating to Egyptian and Babylonian mathematics has made imperative new studies on the relationships of the oriental to the Greek mathematics. Tannery would have been among the first to accord to the orientals that recognition which is their due.

The historical development of the ideas of the infinitesimal is now understood more completely through the recently discovered work of Archimedes on Method. Tannery was probably the first and certainly the most influential in indicating the actual significance of the work of Zeno, particularly its bearing on the development of mathematical ideas.

The long labors of Tannery and Heath, with also Zeuthen and Heiberg, are now reflected in popular treatises on the historical development of Greek mathematics. These can now be based upon the firm foundation of not only dozens of actual Greek treatises but also upon hundreds of supporting documents showing the progress of these ideas in Greece and Europe. The mathematics of medieval Europe does not yet enjoy any similar complete documentation so essential to real understanding. In the enjoyment of the fruits of the labors of

these pioneers in Greek science modern readers should remember these men with gratitude.

L. C. KARPINSKI

Funktionentheorie, Zweiter Teil: Anwendungen und Weiterführung der allgemeinen Theorie. Vierte, verbesserte Auflage. By Konrad Knopp. Sammlung Göschen, No. 703. Berlin and Leipzig, Walter de Gruyter, 1931.

Aufgabensammlung zur Funktionentheorie, Erster Teil: Aufgaben zur elementaren Funtionentheorie. Zweite, verbesserte Auflage. By Konrad Knopp. Sammlung Göschen, No. 877. Berlin and Leipzig, Walter de Gruyter, 1931.

The Göschen collection of texts on mathematics provides a course for the student of the theory of analytic functions which is contained in five of the very convenient pocket size volumes. The two volumes entitled *Funktionentheorie* by Knopp have now appeared in a fourth edition. The second edition of the *Einführung in die konforme Abbildung* by Bieberbach appeared in 1927. The other two volumes contain collections of problems.

The first volume of the fourth edition of the *Funktionentheorie* was reviewed in the American Mathematical Monthly, vol. 38, page 529, by an enthusiastic writer who remarked "There is little doubt but that this is the best monograph on functions of a complex variable yet written." While this superlative statement is a little stronger than those of the reviewers of other editions, it indicates the general attitude of all.

The fourth edition of the second volume of the *Funktionentheorie* shows no extensive changes from the third edition which appeared in 1926. Definitions and proofs have received careful scrutiny and minor improvements are most noticeable in the section on doubly-periodic functions and in the chapter on algebraic functions. The book is divided into two approximately equal parts which treat single-valued functions and multiple-valued functions, respectively. The titles of the chapters are as follows: I. Ganze Funktionen; II. Meromorphe Funktionen; III. Periodische Funktionen; IV. Wurzel und Logarithmus; V. Algebraische Funktionen; VI. Das analytische Gebilde.

The first edition of the first volume of the problems appeared in 1923. The second edition brings the material up to date with references to the latest editions of the texts. Each group of problems contains a specific reference to one of the three volumes which cover the theory. In the first part of the book the problems are merely stated and an asterisk used to designate the more difficult ones. Of 183 problems, 41 are so designated. The second part contains solutions of varying degrees of completeness.

W. R. LONGLEY

Mathematics. By B. B. Low. London, Longmans, Green and Company, 1931. 448 pp.

In contrast to the purpose of this book, the author recalls, in the preface, the toast: "Here's to mathematics, and may it never be of use to anybody." The present work is intended to be useful to those who are engaged in technical work, such as engineering, physics, or chemistry.

The field covered is wide, extending from elementary algebra to differential

equations, including trigonometry, plane and solid analytic geometry, empirical equations, differential and integral calculus, calculus of finite differences, and the harmonic analysis of periodic functions. The extensive use of graphs and illustrations is indicated by the fact that the book contains 409 figures. In addition to many worked examples there are almost 800 exercises. A few of the examples and exercises are taken from the examination papers of Cambridge, the University of London, and the Board of Education, but a very large number are related directly to technical applications.

One might expect almost anything in a book with the title "Mathematics," but when the subtitle "A textbook for technical students" is added, the contents are accurately described. This volume contains practically everything that goes to make up the mathematical equipment of a graduate of one of the best American technical schools. There are, of course, variations. Some topics are treated less extensively and some receive more attention than in our usual curricula, but the reviewer knows of no other single volume which covers so nearly the ground that is commonly regarded as desirable in our technical schools.

W. R. LONGLEY

Science and First Principles. By F. S. C. Northrop. New York, Macmillan, 1931. xiv+299 pp.

In the preface it is stated that the first principles "involved in a given verified theory are those which it takes as primary" and that the author's "task is the purely impersonal and objective one of dissecting the given scientific theories which our technical scientists have verified, to determine what concepts and principles are taken as primary or undefined." The analysis is centered mainly on relativity, quantum theory, and biology. Since the treatment is not convincing to the reviewer, some main conclusions will be presented in the author's own words; usually the conclusions are reached by the author in connection with an elaborate argument. The following quotations are from pages 120, 202, 270, 280, 288, respectively. "We have but to bring these different requirements together to discover that this universe must be constituted not only of the moving microscopic atoms of the traditional atomic theory but also of one large physical macroscopic atom, spherical in shape and hollow in its interior except for its inner field, which surrounds and congests them." [The foregoing conclusion is reached by means of an analysis of relativity.] "Thus we find biological evidence giving us an entirely independent argument for the existence of the macroscopic atomic theory." "In the first place, the macroscopic atom is a primary substance, with a determinate conscious experience. Secondly, were it not present neither nature nor man would exist; all would be flux." "The spherical shell of the macroscopic atom is a tremendous object off at the edge of the whole physical universe." "Once the traditional theory of first principles is supplemented with the addition of this atom, the problem of knowledge is resolved."

R. D. CARMICHAEL

The Emergence of Life. By John Butler Burke. Oxford University Press, 1931. ix+396 pp.

In the subtitle this book is described as "a treatise on mathematical philoso-

phy and symbolic logic by which a new theory of space and time is evolved." The guiding purpose is to clarify the nature of the conception of emergence. The mathematical tool employed is developed from the methods of Boole and particularly from his *Laws of Thought*. The author insists that the methods of Boole belong to "static" thought and he endeavors to show that the Hegelian logic admits of a symbolic treatment which comes into contact with "dynamic" thought. He says (p. 76): "The possibility of carrying the Booleian system beyond the ordinary static or Aristotelian laws of thought has, so far as I am aware, never been previously shown to be possible by algebraic means." In his preface the author says: "I hope that the use of algebraic symbols, without which the reasoning at times is too abstruse for words, will not be regarded as profane in dealing with some of the great and solemn problems under discussion; the language of mathematics being I conceive no less reverent than German, Irish, or Greek; whilst it has the benefit of being universal, pithy, and precise."

R. D. CARMICHAEL

Astronomy. By F. R. Moulton. New York, Macmillan, 1931. xxiii+549 pp. $3.75.

For many years Moulton's *Introduction to Astronomy* enjoyed well-earned popularity as a college textbook. The present book may be considered as a new edition of the former, but the publishers want to emphasize the fact that it is not a mere revision but an entirely new publication. Comparison of the two books shows that this is not an exaggerated statement.

Textbooks on astronomy have tried to keep in step with the rapid development of the younger branches of the science, and the treatment of the solar system is by no means the dominating subject that it was only thirty years ago.

This tendency is evident in Moulton's *Astronomy*. But the treatment of *Stars and Nebulae*, and of *The Sidereal System* is crowded into a smaller portion of the book than is usual in modern textbooks. However clever the exposition of "stellar" astronomy in these chapters is, in a few instances we see the effects of the attempt to cover too much in a limited number of pages.

An obvious danger is to give insufficient attention to some subjects that may be quite important. A rather striking case of this type is the treatment of the spectroscopic method of parallax determination. We find only brief indirect references to this subject on pp. 406 and 414, and the name "spectroscopic parallax" is not used.

Another consequence is that some subjects are covered in so few words that their characterization has become one-sided or even incorrect. As an example we may consider the statement (p. 445) that the spectral-line displacements of the cepheid variables, "interpreted as Doppler effects of radial velocities, do not vary in harmony with elliptic motion." This may be mathematically correct and the briefest possible way to state it. But would it not be preferable to remark how surprisingly well the Doppler shift can be represented by a fictitious elliptic orbit in the majority of known cases?

The influence of Milne's recent criticisms is apparent in the reference to the theory of the interior of stars. Eddington's mass-luminosity relation is almost reluctantly mentioned, and its importance minimized.

The chapters dealing with the dynamics of the solar system are admirable. The author writes as one who knows the beauty of the mathematical treatment of the subject and who thoroughly appreciates the value of the contributions of the great astronomers of past centuries. Much of this admiration and appreciation is conveyed to the student. The clearness of the presentation is the more remarkable considering the fact that this is a decidedly mathematical subject, and that mathematics is, unfortunately, almost completely banished from college textbooks on astronomy.

It is hardly necessary to add that mathematically and historically the book is very accurate. Among other subjects the discussion of the discovery of Pluto (p. 243) deserves being mentioned for its accuracy.

The author's individuality pervades the whole book. This renders it especially interesting to the reader who has sufficient knowledge of the subject to appreciate variations from the presentation by other writers, even if he cannot always agree with the author's emphasis on certain aspects of the science.

The outward form of the book is very attractive: pleasant type, numerous diagrams and reproductions, and some very good star charts. Printing errors are rare, and usually of the innocent type, like *members* for *numbers* and *Betelguese* for *Betelgeuse*.

DIRK BROUWER

De l'Emploi des Droites Isotropes comme Axes de Coordonnées. Nouvelle Géométrie du Triangle. By André Haarbleicher. Paris, Gauthier-Villars et Cie., 1931. vi+76 pp.

This interesting little book serves excellently as an introduction to the analytic geometry of the triangle when the minimum lines through a point are taken as axes of coordinates. The author takes the center of the inscribed circle as origin, developes briefly the necessary formulas, then applies these results to a sequence of problems in the geometry of the triangle, carefully selected to show the value of the method and the types of reasoning employed. The sequence ends with the following problem, which is due to Darboux: Find the locus of a point P such that, among the conics through P and the vertices of a given triangle, there is one to which the normals at these four points concur. Our author remarks "The solution of this problem offers an entirely satisfactory (complet) illustration of the application of isotropic coordinates."

To the student of mathematics, especially if he is preparing to teach in secondary schools, the study of the geometry of the triangle, and allied topics, brings into prominence a number of concepts and methods of interest and permanent value. In this text, he will find an able exposition of a good method of approach to this field from its analytic side. Equally interesting, although the author does not pause even to mention them, are the ideas of projective metric geometry, and of the analytic geometry of the complex plane, to which this method of analysis naturally leads.

American students will doubtless find that the author's frequent introduction of unfamiliar and undefined terms is sometimes troublesome. Otherwise, the text is clear and well written.

C. H. SISAM

Intermediate Mechanics; Statics and Hydrostatics. By D. Humphrey. London, Longmans, Green and Company, 1931. xi+424 pp. $4.50.

This volume is a companion to the author's *Dynamics* and with it constitutes a course in Intermediate Mechanics. The part devoted to statics (342 pages) treats the usual topics, forces at a point and on a rigid body, friction, machines, center of gravity. In addition there are chapters on graphical constructions, bending moments and suspension bridges, and virtual work.

In hydrostatics the work is confined mainly to problems in total pressure, center of pressure, and flotation. One wonders why the author limits the discussion to a perfect fluid inasmuch as the conditions for equilibrium (including the absence of tangential stress) are precisely the same for a viscous fluid. In fact the fundamental property of a fluid is that it does not maintain its shape under the action of tangential stresses however small. This seems preferable to the author's characterization of fluids as media which "offer little or no resistance to change of shape."

The text is clearly written and the problems are excellent and numerous (166 worked and 913 others with answers). The use of calculus is confined almost entirely to the section on hydrostatics. The most serious defect is the entire omission, except in the case of parallel forces, of three-dimensional force systems.

C. A. SHOOK

Analysis des Unendlichen im Psychischen und Physischen. By H. Latzin. Wien, Wilhelm Braumüller, 1931. 32 pp.

The theory of aggregates and its applications in science are treated briefly under the headings: (1) Ordnungen und Mengen, (2) Gestalten, (3) Die "Welt" als Gesamtheit aller Ordnungsstufen, (4) Die "Welt" als Beziehungstotalität einer Ordnung.

R. D. CARMICHAEL

The Logic of Science. By Harold R. Smart. New York, Appleton, 1931. vii+237 pp.

At the beginning of his final chapter the author says: "We have been engaged in defining the content, method, and primary aims of those among the natural sciences which have exerted the greatest influence on philosophical speculation." This indicates more accurately than the title the real character of the book. In his final paragraph the author says of his work, with a confidence which the reviewer does not share, that "the province of science has been clearly defined and its findings have been logically evaluated. . . . We now know in what sense the world of natural science is independent of our apprehension of it as well as in what sense that world is a part or aspect of a larger world of experience. . . . " The importance of the book to philosophers and logicians need not be assessed here; it has little of interest or value to mathematicians, though two out of its eight chapters are devoted to their subject; the natural sciences are treated with somewhat better success, though it can hardly be maintained that the exposition is illuminating even here.

R. D. CARMICHAEL

Philosophy and Modern Science. By Harold T. Davis. Bloomington, Ind., The Principia Press, 1931. xiv+335 pp.

The remarkable concepts of recent physical science are here set forth as the culmination of past experience, and some of their implications are suggested in such a way as to give an insight "into the new philosophy of nature and the metaphysics of modern science." The exposition is non-technical and is about as simple as the character of the material will admit. The major movements in current physics and astronomy are well treated and the progress of ideas is clearly exhibited against a rapidly sketched historical background. The exposition sanely raises the relevant philosophical questions and makes their characteristics stand out with clarity. But these questions are frequently left unanswered—necessarily—in the present state of knowledge and speculation. The book is clear, interesting, and stimulating.

R. D. CARMICHAEL

Vorlesungen über Darstellende Geometrie. By Emil Müller. Band III: *Konstruktive Behandlung der Regelflächen*, bearbeitet von Dr. Joseph Leopold Krames. Leipzig and Wien, Franz Deuticke, 1931. viii+303 pp. M. 35.

The third volume of this well known treatise on descriptive geometry by Emil Müller has been edited and completed by Dr. Krames, professor at the Technische Hochschule at Brünn, and gives a constructive treatment of ruled surfaces from a scientific standpoint, which means argumentation with a rigorous algebraic and analytic basis.

The book is divided into seven chapters with contents as follows: I. Foundations and analytic representations, II. Methods of generating ruled surfaces and theorems connected with them, III. Differential geometry of ruled surfaces, IV. Algebraic geometry of ruled surfaces (Geometrie im Grossen), V.+VI. General and particular ruled surfaces of the third degree, VII. Ruled surfaces of the fourth degree.

Excepting Fiedler's activity at Zürich, culminating in the publication of *Darstellende Geometrie in organischer Verbindung mit der Geometrie der Lage*, Austria has always been the classic country for descriptive geometry with some of the most noted representatives in modern history. The level of instruction in this field at the Austrian polytechnic schools is in a general way above that of many other countries. Here in America for example, descriptive geometry in our schools of engineering is of a very elementary type, as taught in elementary European trade-schools and secondary schools. Somehow there never has been a desire to rise above the elementary level, which, for the purely utilitarian purpose of teaching students merely what is needed in practice is, of course, justifiable.

When reading a treatise on descriptive geometry like Dr. Krames' *Ruled Surfaces*, the beauties and results of a scientific treatment become at once manifest, so that not only the engineer but also the student of mathematics, in particular the geometrician, will profit by the study of such a book. Its value is furthermore increased by historic notes and references, an index, and a list of authors. Typography and figures are excellent.

ARNOLD EMCH

Fourier Integrals for Practical Applications. By George A. Campbell and Ronald M. Foster. (Bell Telephone System Technical Publications, Mathematical Physics, Monograph B-584.) New York, Bell Telephone System, 1931. 177 pp.

This book develops a method for determining transients in physical systems and especially transient effects due to suddenly applied forces in electrical networks. The application of Fourier integrals to problems of this sort is not new but has not been emphasized before, largely because of the difficulty of evaluating the definite integrals involved. The treatment given is rather condensed and without proofs, and would seem to be worthy of more completeness and detail, especially if the method is to be used by any but mathematicians.

In addition to the explanatory text the book contains two tables. The first table, called *A Table of Fourier Integrals*, is primarily a list of paired coefficients, arranged in parallel columns, which are the coefficients appearing in the transforms of the Fourier integral. Many of these have been obtained by transformations and combinations of other pairs previously obtained. A list of rules for combining or operating on pairs already found, to produce new pairs, is given without proof and there is some explanation of the way additional pairs are obtained in special cases. In addition to the use here made of the table, it has considerable interest, per se, as a table of definite integrals. The notation is somewhat elaborate, and at first confusing, and requires a long explanatory note.

The second table contains 85 solutions in "closed form"; that is, without employing infinite series or similar infinite processes, which the authors have found for certain physical systems when one of three definite causes or forces is applied.

The method itself is, briefly, as follows: The two vertical parallel columns of Table I may be referred to, from left to right, as F and G. A cause corresponding to an entry in G is assigned. The mate, F, of this cause is then multiplied by the "admittance" of the system and this product is then sought in Column F. When this is found the corresponding mate, G, represents the effect.

The book represents an ambitious attempt to collect and tabulate a rather complete amount of data in table form which may be used by the physicist or technician without much thought as to the mathematical processes by which the result is arrived at. Since there is always danger in using something which is not completely understood, a somewhat more adequate treatment of the fundamentals underlying the method itself with additional illustrative examples would seem to have been justified and would result in a much wider use of the tables. The compilation of the tables, the checking of the evaluation of the integrals and of the parameter domains represent a vast amount of work. The book stands as a unique contribution to the literature dealing with Fourier integrals and transient problems. One suspects that the earlier work of these authors in this field, which has resulted in the book under review, has inspired the recent interest shown in England in Fourier integrals, self-reciprocal functions, and allied subjects.

J. K. LAMOND

Gewöhnliche Differentialgleichungen. By G. Hoheisel. Zweite, verbesserte Auflage. Sammlung Göschen. Berlin and Leipzig, Walter de Gruyter, 1930. 159 pp.

The first edition of this little book appeared in 1926 and was reviewed in this Bulletin, volume 35, page 136, where it was remarked (by another reviewer) that "as a textbook for the American or English student, it will not be found entirely satisfactory." The improvement in the second edition is not noticeable and the book is still unsuitable as a text, though it may serve satisfactorily as collateral reading.

There are no problems and the examples chosen to illustrate the theory are always of the simplest type, giving sometimes only a vague impression of the situation. It seems particularly unfortunate and misleading to illustrate a singular solution by the example $(y'-xy)(y'-y^2)=0$. The usual assumption that the equation $F(x, y, y')=0$ is irreducible is not made and the example gives an impression of a quibbling triviality.

The book covers a surprising amount of ground and in some places considerable skill is shown in the introduction and brief, though satisfactory, treatment of a topic. In other places, however, considerable carelessness appears, as in the treatment of singular points of linear differential equations on page 101. The equation is taken first in the form

$$p_0(x)y'' - p_1(x)y' - p_2(x)y = q(x),$$

where p_0, p_1, p_2, and q are convergent series in integral powers of x. The point $x=0$ is a singular point if $p_0(0)=0$. Assuming, for simplicity, that $q(x)\equiv 0$ and that $x=0$ is a simple zero of p_0, so that $p_0(x)=x\pi_0(x)$ where $\pi_0(0)\neq 0$, the equation is divided by $\pi_0(x)$ and put in the form $xy''-\pi_1(x)y'-\pi_2(x)y=0$. This equation is then multiplied by x and written $x^2y''-xh_1(x)y'-h_2(x)y=0$, which is the form most convenient for the investigation of regular singular points.

The author does not mention that this is a highly specialized case because $h_2(x)$ must contain x as a factor. The result of the specialization is that one of the two exponents of the equation for the point $x=0$ always vanishes. Ignoring this fact, the treatment proceeds as in the general case including a discussion of conjugate complex roots of the indicial equation, although the only differential equation under consideration has as indicial equation a quadratic with one zero root. Incidentally the terminology "indicial equation" and "exponents" is not used.

While there is no statement in this section that can be called incorrect, the impression is clearly given that a simple zero of p_0 is a singular point of the differential equation leading to the general situation discussed later. The essential fact that p_0 may carry x^2, but no higher power, as a factor is not mentioned. The reader approaching this topic for the first time is not likely to get a correct understanding of it.

The desire to simplify the expositions of each topic as much as possible produced unfortunate results in the case of the singular points mentioned above, but in other places the same method proves very effective and the careful reader may find the book quite helpful in connection with a first study of the subject.

W. R. LONGLEY

The Theory of Spherical and Ellipsoidal Harmonics. By E. W. Hobson. Cambridge, University Press, 1931. vi+500 pp.

This is a comprehensive treatise on the properties of the ordinary and generalized functions of Legendre, with a final chapter on functions of Lamé. The work is all that one would expect from a writer who has himself made many original contributions to the subject, and whose style as an expositor is well known through his text book on real variables. As an up-to-date handbook of spherical harmonics, the new book will undoubtedly be welcomed by both pure and applied mathematicians, and become the standard reference in its field.

PHILIP FRANKLIN

Partielle Differentialgleichungen. By J. Horn. Göschens Lehrbücherei. I. Gruppe, Bd. 14. Berlin and Leipzig, de Gruyter, 1929. 225 pp.

The book is the second edition of the author's *Einführung in die Theorie der partiellen Differentialgleichungen*, which appeared as volume LX in Sammlung Schubert, and contains essentially the same material. Its arrangement, however, has been considerably altered. The three normal forms of linear partial differential equations of the second order (hyperbolic, elliptic, parabolic) are treated successively in the first 6 chapters, the two remaining ones being devoted to non-linear equations with two variables, of the first and second order respectively. Instead of appending a special chapter on partial differential equations in physics, as in the old edition, the author has introduced numerous physical problems and illustrative examples into the text. A discussion of linear integral equations in the earlier pages, sufficiently extensive to render it applicable to later problems, adds greatly to the usefulness of the treatise.

The exposition is clear and logical, and the proofs, while thoroughly rigorous, are always easily followed and enjoyed even by those who seek primarily practical information. The book contains considerably more material than textbooks on advanced calculus, but its size limits it to much less than is to be found in more special treatises on the subject, such as Weber, and Goursat.

HENRY MARGENAU

James Clerk Maxwell; A Commemoration Volume 1831–1931. Essays by J. J. Thomson, Planck, Einstein, Larmor, Jeans, Garnett, Fleming, Lodge, Glazebrook, Lamb. New York, The Macmillan Company, 1931. 146 pp.

This collection of essays, written on the occasion of the celebration of the centenary of Maxwell's birth, gives a vivid impression of the famous man's personality and an idea of the deep influence he has had on the world of natural science. The essays by Thomson and Larmor are not only interesting but very valuable for the sidelights they throw upon the foundation of the Cavendish Laboratory which has since proved so fruitful. There is consolation for us all in the sentence about Maxwell's lectures: "Each term he gave a course of lectures which did not attract as large an audience as they deserved."

F. D. MURNAGHAN

Thermodynamics. By Alfred W. Porter. New York, E. P. Dutton and Co., 1931. 96 pp. $1.10.

This book is one of the series known as *Monographs on Physical Subjects.*

It gives a compact and accurate statement of the modern status of the subject in hand, paying especial attention to the logical and historical foundations. It is recommended to those who want a brief treatment of this important branch of physics.

C. A. SHOOK

Collected Geometrical Papers, Part II. By Syamadas Mukhopadhyaya. Calcutta, University Press, 1931. 139–295 pp.

Part I of the Collected Papers was published in 1930, and reviewed in this Bulletin, vol. 36 (1931), p. 614. In Part II the pagination continues, and the make-up is the same as that of Part I. It contains two recent essays on plane topology which appeared in the Mathematische Zeitschrift in 1931 and the Tôhoku Mathematical Journal in 1931, respectively, and seven on parametric representation of curves in n-space, all but one of which (the Griffiths Memorial Prize Essay of 1910) were published in the Bulletin of the Calcutta Mathematical Society from 1909 to 1915. The argument and points of view of the papers on topology are similar to those in Part I. Only elementary methods are employed, but with striking originality and richness in new results. Most of these concern cyclic and sextactic points on continuous ovals.

The other essays are on the differential geometry of analytic curves in a euclidean n-space. Properties are expressed in terms of determinants of derivatives of various orders of the coordinates as to the parameter.

The first intrinsic parameter is the arc length. The second is the projection of the area of the triangle formed by three points which approach coincidence on the curve, summed over the interval of integration, etc. A curve in S_n has n such intrinsic parameters. They are independent of the coordinates chosen and of the parameter. Any $n-1$ independent equations connecting these parameters will determine a curve in S_n, intrinsically. The generalized idea of curvature, spheric of osculation, quadric of osculation etc. can now be expressed. The results in the case of plane curves are compared with those obtained by projective differential geometry. The same ideas are then extended to curves in S_n. At times the amount of machinery necessary seems a bit bewildering, but one is soon consoled by an unexpected general theorem evolving from the maze of formulas. The various kinds of singular points and the associated parametric representation in series are treated in great detail. The Papers contain a powerful weapon with which to attack metric problems on analytic curves of hyperspace.

VIRGIL SNYDER

NOTES

Beginning with the 1932 volumes, the Jahrbuch über die Fortschritte der Mathematik and the Revue Semestrielle des Publications Mathématiques will be combined into a single periodical, edited jointly by the Prussian Academy of Sciences and the Amsterdam Mathematical Society. It will appear in two parts: one, the successor to the Revue Semestrielle, will appear every two months and will consist only of a list of titles of books and memoirs marked with the index signs used by the Revue Semestrielle and the Jahrbuch, and the other, the successor to the Jahrbuch, will appear semi-annually, and will contain complete abstracts. This journal is published by de Gruyter, of Berlin.

A new journal, the Ergebnisse eines mathematischen Kolloquiums, has been founded under the editorship of Karl Menger, with the assistance of K. Gödel and G. Nöbeling. It is published by Teubner.

A new Japanese journal, of interest to mathematicians, has appeared, the Journal of Science of the Hiroshima University, series A (mathematics, physics, chemistry).

The British Association for the Advancement of Science will hold its annual meeting for 1932 at York, August 31–September 7, under the presidency of Sir J. Alfred Ewing.

The Lazzaro Fubini prize of the Unione Matematica Italiana has been awarded to Giuseppe Scorza.

The Franklin Institute has awarded a Cresson medal to Professor P. W. Bridgman, of Harvard University.

Juan de la Cierva has been awarded the Guggenheim gold medal for the promotion of aeronautics, for his work in the development of the theory and practice of the autogyro.

Professor R. A. Millikan, of the California Institute of Technology, has been awarded the Roosevelt medal of the Roosevelt Memorial Association.

Among members recently elected in the division of science of the Bavarian Maximilian Order for Science and Art are Dr. Walther von Dyck, professor of mathematics, and Dr. Arnold Sommerfeld, professor of theoretical physics, both of the University of Munich.

Paul Painlevé has been appointed Minister of the Air in the Edouard Herriot cabinet.

Professor Paul Stroobant, of the University of Brussels, has been elected correspondent of the Paris Academy of Sciences in the section of astronomy.

Dr. W. L. Bragg, professor of physics at the Victoria University of Manchester, has been elected a member of the Athenaeum, London, under a provision which empowers the annual election by the committee of a certain num-

ber of persons of distinguished eminence in science, literature, the arts, or for public service.

Sir Arthur Eddington has been elected a foreign member of the Hungarian Academy of Sciences.

The University of Manchester has conferred honorary doctorates on Sir James Jeans and Lord Rutherford.

The honorary doctorate of the mathematical faculty of the University of Göttingen has been conferred on Lord Rutherford.

McGill University has conferred an honorary doctorate on Professor A. N. Whitehead.

The following have been elected members of the American Academy of Arts and Sciences: C. R. Adams, A. A. Bennett, Einar Hille, and D. V. Widder.

Professor Marston Morse, of Harvard University, has been elected a member of the National Academy of Sciences.

M. F. Deuring, of Leipzig, has been awarded a Sterling fellowship at Yale University.

H. Beghin has been appointed to the professorship of physical and experimental mechanics at the University of Paris, as successor to the late Gabriel Koenigs.

Dr. Rudolf Carnap, of the University of Vienna, has been appointed associate professor of philosophy of the natural sciences at the German University at Prague.

Cambridge University has conferred the title of Stokes lecturer in mathematics on Dr. P. A. M. Dirac.

At Cambridge University, Mr. N. F. Mott, of Gonville and Caius College, has been appointed university lecturer in the faculty of mathematics.

Professor Wilhelm Plaschke, of the University of Hamburg, lectured in early April, 1932, at the National University of Peking and Tsing Hua University, Peiping, China, on topological problems in differential geometry, problems in integral geometry, and linear algebra.

On April 19, 1932, Professor Robert Courant of Göttingen delivered the first William Lowell Putnam Lecture of 1932 at Harvard University on the subject *Hyperbolic partial differential equations and wave mechanics.*

Professor Arnold Dresden of Swarthmore College gave a lecture at Oberlin College, April 25, 1932, on *Logic, mathematics and reality* under the auspices of the departments of philosophy and mathematics.

On April 27, 1932, Professor R. L. Moore, of Texas, delivered the second William Lowell Putnam Lecture of 1932 at Harvard University on the subject *The foundations of point set theory.*

Professor Werner Heisenberg, of the University of Leipzig, will take part in the fifth annual symposium in theoretical physics at the University of Michigan, to be held June 27–August 19, 1932.

Dr. Hassler Whitney, National Research Fellow, will conduct a seminar in Metric Spaces at Harvard University during the first term of 1932–33.

Dr. Beulah M. Armstrong, of the University of Illinois, has been promoted to be associate in mathematics.

Dr. H. W. Bailey has been promoted to an assistant professorship at the University of Illinois.

Assistant Professor B. H. Brown has been promoted to a professorship of mathematics at Dartmouth College.

Dr. W. E. Byrne has been appointed associate professor of mathematics at the Virginia Military Institute.

J. P. DenHartog, chief of the dynamics section of the Westinghouse Electric and Manufacturing Company, has been appointed assistant professor of applied mechanics at Harvard University.

Dr. J. L. Dorroh has been appointed research assistant at Princeton University.

Dr. J. M. Earl has been appointed professor of mathematics at the Municipal University of Omaha.

Dr. O. J. Farrell has been appointed assistant professor of mathematics at Union College.

Assistant Professor J. H. Fithian has been promoted to an associate professorship of mathematics at the Newark College of Engineering.

Dr. W. W. Flexner has been promoted to be associate in mathematics at Bryn Mawr College.

Dr. H. H. Germond has been appointed assistant professor of mathematics at the University of Florida.

Professor F. J. Gerst, of St. Louis University, has been appointed professor and head of the department of mathematics at Loyola University, Chicago.

Assistant Professor R. F. Graesser, of the University of Arizona, has been promoted to an associate professorship of mathematics.

R. A. Hefner has been promoted to an assistant professorship at the Georgia School of Technology.

Assistant Professor L. D. Hemenway has been promoted to an associate professorship at Simmons College.

Dr. C. G. Jaeger has been appointed professor of mathematics at Pomona College.

Dr. Cornelius Lanczos, on leave from the University of Frankfort, who has been visiting professor of mathematics at Purdue University during 1931–32, has received a permanent appointment at the latter institution as professor of mathematical physics. Professor Lanczos will be in residence during the first semester only.

Mr. H. G. Lieber has been appointed assistant professor of mathematics at Long Island University.

Dr. W. O. Menge has been promoted to an assistant professorship of mathematics at the University of Michigan.

Dr. J. E. Merrill has been appointed curator of astronomy at the Buffalo Museum of Science.

Dr. J. E. Powell has been promoted to an assistant professorship at Michigan State College.

Assistant Professor E. L. Thompson has been promoted to an associate professorship of mathematics at the Texas Technological College.

Dean Frederick Wood, of Hamline University, has been appointed professor of mathematics at the University of Nevada.

Dr. Oscar Zariski has been promoted to an associate professorship of mathematics at Johns Hopkins University.

Dr. Leo Zippin has been appointed research assistant at Princeton.

The following appointments to instructorships in mathematics are announced:
Brooklyn College of the City of New York, Dr. J. M. Feld, Dr. L. S. Kennison, Mrs. Jennie P. Kormes;
Harvard University, Dr. Wladimir Seidel to a Benjamin Peirce Instructorship and P. A. Adams, T. L. Downs, J. S. Frame, A. S. Galbraith, Z. I. Mosessон, L. T. Moston, F. H. Steen to Part Time Instructorships.
University of Minnesota, Mr. C. H. Fischer;
Northwestern University, Dr. W. J. Trjitzinsky;
Ohio State University, Dr. L. E. Bush;
Pennsylvania State College, Dr. T. C. Benton, Dr. Beatrice L. Hagen;
Princeton University, Mr. M. M. Flood;
Stanford University, Mr. F. A. Butter;
University of Wisconsin, Mr. K. W. Wegner;
Yale University, Mr. W. R. Church.

Paul Barbarin, of the Ecole Spéciale des Travaux Publiques, Paris, died September 28, 1931, at the age of seventy-three.

Professor Michele Cantone, of the University of Naples, died March 25, 1932.

Luigi Donati, professor emeritus of technical physics at the University of Bologna, died March 7, 1932.

Professor A. Ostenfeld, of the University of Copenhagen, died September 21, 1931.

Professor Giuseppe Peano, of the University of Turin, died April 20, 1932, at the age of seventy-three.

Professor Friedrich Schur, of the University of Breslau, is dead.

Professor Giuseppe Vitali, of the University of Bologna, died March 7, 1932, at the age of fifty-six.

Dr. L. A. Bauer, director emeritus of the Department of Terrestrial Magnetism of the Carnegie Institution of Washington, died April 12, 1932, at the age of sixty-seven.

Professor C. G. Simpson, of the department of mathematics of the School of Engineering, Milwaukee, died February 5, 1932, at the age of sixty.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THIS SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

159. Dr. O. K. Bower: *Applications of an abstract existence theorem to both differential and difference equations.*

A functional equation of the form $f=g+Sf$, in which g is a known function and S a suitable functional operator, has for some time been used to prove existence theorems for integral and differential equations (Max Mason, *Selected topics in the theory of boundary value problems of differential equations*, The New Haven Mathematical Colloquium, 1906, p. 173). This paper considers an equation of this form with modified restrictions on the function g and operator S and proves an abstract existence theorem which can be applied directly to proving the existence of solutions of integral, finite integral, q-integral, differential, difference, and q-difference equations having relatively simple solutions in the neighborhood of infinity. The principal novelty of this treatment lies in making all of the six cases depend on a single abstract theorem, which is possible because of the inherent similarities in the operations of differentiating, differencing, and q-differencing. (Received May 11, 1932.)

160. Dr. O. K. Bower and Mr. J. D. Grant: *A system of simultaneous bilinear functional equations.*

This paper considers the system of simultaneous bilinear functional equations S: $u_j(x+y)=\sum_{i=1}^{n} a_{i,\,n+j+1-i}u_i(x)u_{n+j+1-i}(y)$, $j=1, 2, \cdots, n$; $a_{i,\,n+j+1-i}=a_{i,\,j+1-i}$, $u_{n+j+1-i}=u_{j+1-i}$, $i\leqq j$, in which the constant coefficients $a_{i,\,n+j+1-i}$ are such that there is a solution consisting of n linearly independent and continuous functions. This system of equations is a generalization of the cosine and sine, hyperbolic cosine and hyperbolic sine addition formulas ($n=2$), and has likewise as a special case when $n=3$ the functions u_1, u_2, u_3 discussed by Mr. L. E. Ward (American Mathematical Monthly, vol. 34 (1927)). It is shown that there are as many "normal" forms for S for each value of n as the number of divisors of n, unity and n included, and that the number of entirely arbitrary constants in the general solution of each of these forms is equal to n. (Received May 11, 1932.)

161. Professor Alonzo Church: *A set of postulates for the foundation of logic.* Second Paper.

In a recent paper (Annals of Mathematics, vol. 33 (1932), pp. 346–366) the author proposed a set of postulates for logic, which, it was hoped, would avoid

some of the complications of *Principia Mathematica*. In the present paper it is found necessary to add four new postulates to this set, and it is also observed that Postulates 2 and 24 of the original set are not independent and may be omitted. Some of the first consequences of the revised list of postulates are developed. It is proved that, of all propositional functions equivalent to ϕ, there is one, called the *completion* of ϕ, for which the range of the independent variable is greatest. In terms of the notion of the completion of a propositional function, theorems are proved which validate certain forms of the method of reductio ad absurdum. A program is outlined by which it is hoped to obtain a theory of positive integers and a theory of real numbers. The possibility is discussed of making a metamathematical proof, in the sense of Hilbert, that these postulates cannot lead to a contradiction, and it is pointed out that the recent proof by Kurt Gödel (Monatshefte für Mathematik und Physik, vol. 38 (1931), pp. 173–198) of the impossibility of such a proof of freedom from contradiction does not apply to this case. (Received May 16, 1932.)

162. Mr. J. G. Deutsch: *Functions satisfying a generalized Lipschitz condition.*

In this paper functions are defined on a set E which satisfy a Lipschitz condition relative to sets whose measures approximate that of E. Such a function is said to have the L-property relative to E. A necessary and sufficient condition is established that a function have the L-property. For example, functions which possess finite partial derivatives have the L-property. Transformations whose components have the L-property are considered and certain of the classical results on transformations are shown to be valid for these transformations. It is proved that a function of two or more variables which is monotone on a set is totally differentiable relatively to that set almost everywhere in the set. It is a consequence of this result that functions of bounded variation in the sense of Arzelà or in the sense of Hardy and Krause are totally differentiable almost everywhere. (Received May 13, 1932.)

163. Mr. C. A. Lovell: *On certain associated metric spaces.*

For every Einstein space of non-zero scalar curvature there exists an associated space whose fundamental tensor is the Ricci tensor for the Einstein space. This paper considers, among other things, the space defined by the assignment of a value to the curvature of the associated space. The idea of associating surfaces in the manner mentioned above is extended to produce an ordered sequence of surfaces, the fundamental tensor of each member of the sequence being the Ricci tensor of the surface which precedes it in the sequence. The members of the sequence are the original surface, the first associated Ricci surface, the second associated Ricci surface, etc. Among the interesting special cases discussed are those in which the first associated space is flat or has constant scalar curvature. Included in one of these special cases are found to be all surfaces applicable to a minimal surface. The equation of condition for these cases is integrated, giving the most general examples of such surfaces. The case where the mth associated Ricci surface is the original surface is also discussed. (Received May 24, 1932.)

164. Professor A. A. Albert: *Non-cyclic normal division algebras of degree and exponent four.*

The author recently proved the existence of non-cyclic normal division algebras in a paper to be published in the June, 1932, issue of this Bulletin. The algebras constructed were direct products of (cyclic) algebras of order four and hence did not have a structure very different from cyclic algebras. The present paper presents certain normal division algebras of order sixteen not expressible as direct products of algebras of order four and proves them to be non-cyclic. Thus the first example in the literature is given of normal division algebras with a structure proved to be more complicated than that of the simplest type, the cyclic (Dickson) algebras. (Received May 26, 1932.)

165. Miss Esther McCormick: *On reduction of space Cremona involutions.*

Every involutorial transformation of the second degree can be reduced. Let T_m be any one of the general monoidal involutions with stars coincident and corresponding in homology. If an S_n exists such that $S_n^{-1}T_mS_n$ is a collineation, then $n>m$ and $n\geqq am$, where a depends on an F curve of S_n. For every m there are infinite sets of T_m's which can be reduced to collineations by an S_n with $n<m$. (Received June 3, 1932.)

166. Professor H. R. Brahana: *Groups $\{S, T\}$ whose commutator subgroups are abelian.*

This paper gives a complete determination of the groups generated by two operators, T of order 2 and S of prime order p, whose commutator subgroups are abelian. (Received June 14, 1932.)

167. Professor H. R. Brahana: *Prime-power abelian groups generated by a set of conjugates under a special automorphism.*

The abelian groups H which are commutator subgroups of groups $\{S, T\}$ have, among others, the following properties: H admits an isomorphism U of order p, the order of S; H is generated by the conjugates of one of its operators under powers of U, of which no more than $p-1$ are independent; H contains no operator of order different from p invariant under U. If H is of order q^n, where q is a prime distinct from p, then $n=\alpha(k_1m_1+k_2m_2+\cdots+k_jm_j)$ and H has $k_i\alpha$ independent generators of order q^{m_i}, where α is the exponent to which q belongs, mod p, and $k_1+k_2+\cdots+k_j\leqq(p-1)/\alpha$. If H is of order p^n, then $n=k_1m+k_2(m-1)$ and H has k_1 and k_2 independent generators of order p^m and p^{m-1} respectively, where $k_1+k_2=p-1$ and $m\geqq1$; or $n=k+2$ and H is of type 2, 1, 1, $\cdots$, where $k<p-2$. (Received June 14, 1932.)

168. Dr. Leonard Carlitz: *Note on polynomials in a Galois field.*

In this paper are proved a number of miscellaneous results concerning polynomials in a single indeterminate with coefficients lying in a Galois field. The results are of two kinds. The first are formulas involving certain divisor and generalized totient functions. The second consist of expressions for the

L.C.M. of the polynomials of given degree, the product of all such polynomials, and the product of those that are not divisible by any ρth power ($\rho>1$). (Received June 16, 1932.)

169. Dr. Leonard Carlitz: *On the representation of a polynomial in a Galois field as the sum of an even number of squares.*

In this paper simple expressions are obtained for the number of solutions of $\alpha F=\alpha_1X_1^2+\beta_1Y_1^2+\cdots+\alpha_sX_s^2+\beta_sY_s^2$, where(A): $\alpha=\alpha_1+\cdots+\beta_s\neq 0$, $(\alpha_1+\beta_1)\cdots(\alpha_s+\beta_s)\neq 0$; X_i, Y_j of degree k, F of degree $2k$; (B): $\alpha_1+\cdots+\beta_s=0$; X_i, Y_j of degree k, F of degree $f<2k$. The results hold for all positive s, the form depending essentially on whether $(-1)^s\alpha_1\cdots\beta_s$ is or is not a square in the underlying Galois field. (Received June 16, 1932.)

170. Dr. Leonard Carlitz: *On the representation of a polynomial as the sum of an odd number of squares.*

By means of the results in a paper on an even number of squares (this Bulletin, vol. 38 (1932), abstract 169), the corresponding problems are here treated for the case of an odd number of squares. The results are different from those for the former problem; they involve the sums $\sigma_t=\sum(A/F)$, summed over all A of degree t, (A/F) being Dedekind's symbol of quadratic residuacity. For the case F quadratfrei, the results are particularly simple. (Received June 16, 1932.)

171. Professor R. D. Carmichael: *Systems of linear difference equations and expansions in series of exponential functions.*

The principal purpose of the first part of this paper is to establish existence theorems for the solution of the system $\sum_{i=1}^{n}e_{\nu i}g_i(x+a_{\nu i})=\phi_\nu(x)$, $(\nu=1, 2, \cdots, n)$, of generalized non-homogeneous linear difference equations with constant coefficients, the given functions $\phi_\nu(x)$ being integral functions and the dependent functions $g_i(x)$ being required to be integral functions. The results obtained are applied in the second part to the rather remarkable problem of the simultaneous expansion of n given integral functions in composite power series. Finally the third part of the paper is devoted to the theory of a class of remarkable expansions in series of exponential functions, generalizing the theory of Fourier series in an interesting direction. Whereas the basic region of convergence of Fourier series is a segment of a straight line, these new series, apart from certain particular cases, have certain polygons in the complex plane as their basic regions of convergence. (Received June 18, 1932.)

172. Dr. A. E. Currier: *The fundamental theorems on second-order cross partial derivatives.*

In a paper submitted to the Transactions of this Society we have proved the following theorems. Theorem 1. *Let $f(x, y)$ be a function of Baire defined on an open region R. Let the first partial derivative f_x exist on R. Let A be a point set on which the partial derivatives f_{xx} and f_{xy} exist almost everywhere. Then the following*

approximate limit, approx-$\lim_{\lambda\to 0}(1/\lambda^2)[(x+\lambda, y+\lambda)-f(x, y+\lambda)-f(x+\lambda, y)+f(x, y)]$, *exists and equals* f_{xy} *almost everywhere on* A. Theorem 2. *Let* $f(x, y)$ *be defined on an open region* R. *Let* f_x *and* f_y *exist on* R. *Let* A *be a set on which* $f_{xx}, f_{xy}, f_{yx}, f_{yy}$ *exist almost everywhere. Then* $f_{xy}=f_{yx}$ *almost everywhere on* A. We can replace f_x by $D_x f$ in the above theorems, where $D_x f$ is the upper or lower right or left hand first partial derivative of f with respect to x. Theorems 1 and 2 can now be generalized so as to avoid all reference to the existence of first partial derivatives. (Received June 9, 1932.)

173. Mr. H. G. Russell and Professor J. L. Walsh: *On the convergence and overconvergence of sequences of polynomials of best simultaneous approximation to several functions analytic in distinct regions.*

Let M be an arbitrary closed limited point set (not a single point) of the z-plane whose complement K (with respect to the entire plane) is connected and regular in the sense that the Dirichlet problem can be solved for K. Let $w=\omega(z)$ be a function which maps K onto the exterior of the unit circle in the w-plane so that $z=\infty$ corresponds to $w=\infty$. Let C_R denote the transform in the z-plane of $|w|=R$, $R>1$, under the mapping function $\omega(z)$. Then it is proved that (1) if the function $f(z)$ is analytic on and within C_R, there exist polynomials $P_n(z)$ of respective degrees $n=1, 2, \cdots$, such that the inequalities (a) $|f(z)-P_n(z)|\leq N/R^n$, z on M, where N is independent of n and z, are valid for z on M; (2) if there exist polynomials $P_n(z)$ of degrees $n=1, 2, \cdots$, such that (a) holds for z on M, the sequence $\{P_n(z)\}$ converges uniformly on any closed point set interior to C_R. Applications are made to sequences of polynomials of *best approximation*. (Received May 25, 1932.)

174. Professor J. P. Ballantine: *A proof of E. H. Moore's equivalence.*

Moore's equivalence states that the existence of a solution of a system of linear equations is equivalent to a certain implicational proposition. His proof holds in any number system where division except by zero is always uniquely possible. In order to orthogonalize the columns of the matrix of the coefficients, he has to assume that if any column, considered as a vector, has a norm zero then every coefficient in that column vanishes. In a system of residues modulo p, important cases arise when this hypothesis is not satisfied, and yet the equivalence holds. A proof is given which does not require the hypothesis. It is pointed out that the fundamental lemma of the calculus of variations is a special case of Moore's equivalence. (Received June 20, 1932.)

175. Mr. Amos Black: *Types of involutorial space transformations associated with certain rational curves.*

Let there be given a pencil of surfaces $|F_n|$ which contains a rational curve r_m as an $(n-2)$-fold basis curve. If the surfaces of the pencil are projective with the points of the curve, a general line of the complex of secants of r_m will intersect the associated surface in two points P, P' not on r_m. These two points

are a pair in involution. The admissible cases other than the straight line, n general, m general, and the pencil of quadrics are: $n=3$, $m=2, 3, 4, 5$; $n=4$, $m=2, 3$. When $n=4$, $m=3$ the involution has a new kind of singularity, an infinity of parasitic lines which are the generators of a ruled surface. (Received June 22, 1932.)

176. Dr. A. L. Foster: *On general Kronecker-(integer)-synthesis of disciplines.*

The present communication establishes certain foundations basic in any integer-synthesis, or analysis, in the Kronecker sense; i.e., in the construction (or reduction) of mathematical disciplines as integer-disciplines. (Cf. J. Stein, Mitteilen der Mathematischen Gesellschaft in Hamburg, vol. 7, No. 2, in which the disciplines (1) rational arithmetic, and (2) (Gauss) complex number arithmetic are Kronecker-reduced in a satisfactory way.) The construction and study of (1) equivalence-functions (e.g., the "identical," $n=n$ of arithmetic; or again, for example, the "derived" equivalence, $\asymp$, which says $m/n \asymp m/n$ if $mn=mn$), and (2) equivalence-conserving functions (essentially functions which, for equivalent arguments, take on equivalent values) are considered. In (1) a general and powerful method of mathematical construction of equivalence functions is discovered, in a covering process, associated with Abelian systems (not necessarily groups) of transformations. The development proceeds from an extremely primitive axiomatic. ("Objects," "characteristics" (or "Merkmale"), "basic equivalence function.") As yet merely certain simple special "denumerable" cases opened up by this theory have been considered. Besides familiar disciplines, other cases present themselves. To what extent this type of investigation will also prove fruitful in the "non-denumerable" case cannot as yet be said. (Received June 23, 1932.)

177. Mr. Ralph Hull: *The numbers of solutions of congruences involving only k-th powers.*

Following preliminary theorems concerning the numbers of solutions of general congruences of the type $\sum_\nu a_\nu x_\nu^k \equiv a \pmod n$, $(\nu=1, \cdots, s;\ k \geqq 1;\ s \geqq 1;\ a_1, \cdots, a_s, a$ and n any integers), the congruence $\sum_\nu x_\nu^k \equiv a \pmod p$, p a prime, is considered. This congruence has the same number of solutions as the congruence $\sum_\nu x_\nu^m \equiv a \pmod p$, where m is the greatest common divisor of k and $p-1$, and formulas are found for the numbers of solutions for $m \geqq 2$. These are of the nature of recursion formulas and reduce the problem for given m and p to the determination of m^2 integers for obtaining which a method is indicated. For $m=5$, the general formulas, together with a special discussion, yield expressions for the coefficients of the reduced form of the quintic resolvent of the cyclotomic equation $x^{p-1}+\cdots+1=0$, $p=5h+1$, in terms of an integral solution of the simultaneous equations $x^2+25y^2+25z^2+125w^2=16p$, $y^2+yz-z^2=xw$. These equations have exactly eight distinct solutions in integers for each prime $p=5h+1$. Sufficient conditions on s are indicated in order that the congruence $\sum_\nu x_\nu^k \equiv a \pmod n$ may have a solution for every a and n. (Received June 24, 1932.)

178. Professors W. D. MacMillan and Walter Bartky: *Permanent configurations in the problem of four bodies.*

If there exists a configuration in the problem of n bodies in which the attraction on each one of the bodies due to all of the others is directed toward the center of gravity and is proportional to the distance of the body from the center of gravity, then the equations of motion of the n bodies reduce to the equations of motion of the two-body problem. Therefore Keplerian motion of the n bodies, in which the configuration is preserved, is possible. The Lagrangian solutions for three bodies, Moulton's straight line solution for n bodies, and other particular configurations come under this general theorem. Hence the problem reduces to finding such configurations. An analysis of the configurations of the four-body problem is given. Quadrilaterals are classified: convex, concave, and straight lines. By Moulton's theorem, any four masses in any assigned order can be arranged uniquely on a straight line so as to satisfy the necessary conditions. For any four masses and assigned order there exists at least one convex quadrilateral (uniqueness not proved) that satisfies the conditions. For concave quadrilaterals the same theorem holds provided the mass ratios lie within certain bounds. Within certain sharp restrictions the quadrilateral can be taken at random, and the masses are, with a single exception, uniquely determined. These methods extend to the case of more than four bodies. (Received June 23, 1932.)

179. Professor R. D. Carmichael: *Note on triple systems.*

The theorem of Skolem (Norsk Matematisk Tiddsskrift, vol. 13 (1931): pp. 41–51), proved also by Hasse (ibid., pp. 105–107), concerning the existence of triple systems having the property that the presence of the triples (abu), (bcv), (ucx) in such a system implies the presence of the triple (avx) in the system is easily shown to be essentially a special case of the existence theorem for the finite geometries of Veblen and Bussey: the triples in such a system are the sets of collinear points in the geometry $PG(k, 2)$ where k is any positive integer. (Received June 25, 1932.)

180. Professor W. H. McEwen: *On the approximate solution of linear differential equations with boundary conditions.*

This paper is concerned with the approximate representation of the solution of a linear differential equation of the mth order $L(y)=R(x)$ with m linearly independent two-point boundary conditions $U_i(y)=h_i$ $(i=1, 2, \cdots, m)$, by means of polynomials $P_n(x)$ defined so as to minimize an expression involving the boundary conditions as well as the integral of the rth power of $|L(P_n)-R|$. (Received June 27, 1932.)

181. Professors E. R. Hedrick and W. M. Whyburn: *An application of the Dedekind cut notion to integration.*

The authors introduce a notion of *cut*, analogous to the Dedekind number cut, in the class of all simple functions on the real interval $a \leqq x \leqq b$ (see this Bulletin, vol. 38, page 123, for definition of *simple function*). Two classes of

functions are defined on a basis of this cut. One of these classes contains all bounded functions that are Riemann integrable and is contained in the class of all Lebesgue integrable functions. It is shown to be identical with the class of all functions that are continuous on the given interval minus a set of points of measure zero. The second class contains all functions (bounded or unbounded) that are integrable in the Lebesgue sense. Other developments of the notion of *cut* are given. (Received June 27, 1932.)

182. Professor E. V. Huntington: *New sets of independent postulates for the algebra of logic, with special reference to Whitehead and Russell's Principia Mathematica.*

This paper contains two sets of postulates for Boolean algebra in terms of $(K, +, ', =)$, and the following set of nine postulates for Section A of the *Principia* in terms of $(K, +, ')$ alone. *Group A.* 1.71. If a and b are in K, then $a+b$ is in K. 1.7. If a is in K, then a' is in K. *Group B.* There exists in K a subclass T having the following five properties: 1.1. If a is in T and $a'+b$ is in T, then b is in T. 1.2. $(a+a)'+a$ is always in T. 1.3. $b'+(a+b)$ is always in T. 1.4. $(a+b)'+(b+a)$ is always in T. 1.6. $(b'+c)'+[(a+b)'+(a+c)]$ is always in T. *Group C.* If T is a subclass having these five properties, then T has the following further properties: 1.8. If $a+b$ is in T, then a is in T or b is in T. 1.9. If a' is in T, then a is not in T. The first seven postulates correspond to the "formal" primitive propositions in the *Principia*; the last two correspond to two "informal" statements in the *Principia*. Through postulates 1.8 and 1.9 the "formal" and "informal" definitions of $a=b$ (as an "element" and as a "relation") become equivalent. The consistency and independence of the nine postulates are established by usual methods. (Received June 25, 1932.)

183. Professor Nathan Altshiller-Court: *The Apollonian spheres of a tetrahedron.*

Consider the four spheres having for centers the vertices of a tetrahedron (T) and having their radii proportional to the respective altitudes of (T). The six spheres of similitude of these four spheres, taken in pairs, are, by definition, the *spheres of Apollonius* of the tetrahedron. Apollonian spheres exhibit many analogies to similarly defined Apollonian circles of a triangle. The two bisecting planes of a dihedral angle of (T) meet the opposite edge in two diametrically opposite points of the Apollonian sphere having its center on the latter edge. The centers of the six Apollonian spheres lie in the same plane, the *Apollonian plane* of the tetrahedron. The six Apollonian spheres have two points in common, but, contrary to the case of the Apollonian circles, these two points are not necessarily real. The six radical planes of the Apollonian spheres with the circumsphere of (T) have a point in common. The harmonic pole of the Apollonian plane with respect to (T) and the centroid of (T) are two isogonal conjugate points of (T). The twelve points determined by the pairs of bisecting planes of the dihedral angles of a tetrahedron upon the respectively opposite edges form a desmic system of tetrahedrons. (Received July 5, 1932.)

184. Mr. G. P. Wadsworth: *Systems of three linear partial differential equations of the third order.*

This paper considers the solution of a system of three linear homogeneous partial differential equations of the third order, using an extension of the method of P. Franklin and C. L. E. Moore (Journal of Mathematics and Physics, vol. 9, No. 1, 1930, p. 22) for two partial differential equations of the second order. Two canonical forms are found to which the original equations can be reduced, and it is shown how the three canonical forms which E. P. Lane (The Tôhoku Mathematical Journal, vol. 33, Nos. 1, 2, p. 12) found from geometrical considerations may be reduced to these two forms. Thus the solution of a system of third order linear partial differential equations reduces to the solution of one of the two canonical forms of which it is a consequence. In the case of one of the canonical forms, the solutions are linear combinations with arbitrary constant coefficients of eight or fewer definite functions. In the case of the other canonical form, the solutions are certain linear combinations of arbitrary constants, an arbitrary function, and its derivatives, where the coefficients of these functions are definite functions which depend upon the original system of equations. (Received July 1, 1932.)

185. Professor W. M. Whyburn: *A classification of the critical sets for functions.*

In a former paper (this Bulletin, vol. 35, pp. 701–708) the author studied the *critical sets* for continuous real functions of n real variables. The functions were assumed to have continuous first partial derivatives. The present paper separates the critical sets of functions into five classes on the basis of the complimentary domains of certain closed point sets. A notion of *order* of a critical set is introduced and relations are established between the five classes. Four of the classes correspond, in a general way, to the ordinary minimal, maximal, minimax, and flex points and degenerate into these under more restrictive hypotheses. The notions of *critical point* and *critical set* are extended to continuous functions which do not have first partial derivatives, and many of the former results are shown to hold for this less restricted class of functions. (Received July 6, 1932.)

186. Professor E. T. Bell: *Diophantine equations from algebraic invariants and covariants.*

Few Diophantine equations of degree higher than the second in more than two variables have been *completely* solved in integers; see Dickson, this Bulletin, vol. 27, 1921, pp. 312–319, also *History of the Theory of Numbers*, vol. 2. In the present paper the following method is applied to obtain *complete* solutions of some such equations. Necessary and sufficient conditions that a given general quantic have a repeated factor, or that it be totally reducible, or that it have a factor of given degree, etc., are expressed by the *identical* vanishing of certain covariants. Equivalent conditions can be obtained by the method of undetermined coefficients. Comparing the two sets, we get a *complete parametric solution* of a Diophantine system. From this solution, *all integer solutions*

are obtained by selecting from the sets of values, rational and irrational, of the parameters *all* those which give integer points on the locus represented by the system. The method is applied in detail to the general quartic developable in homogeneous coordinates in 3-space, and explicit formulas in terms of integer parameters are given for all integer points on the surface. (Received July 7, 1932.)

187. Mr. H. W. Raudenbush, Jr.: *Differential fields and ideals of differential forms.*

Fields in which each element has a unique derivative which is also an element have been considered by R. Baer and used by O. Ore. In this paper, the author shows that extensions of such differential fields to differential fields satisfy theorems analogous to Steinitz' theorems (Journal für Mathematik, vol. 137 (1910), p. 167; theorem 2, p. 293; 3, p. 299; 4, p. 300) on the degree of transcendency of extensions of algebraic fields. A differential form is defined as a polynomial in indeterminates and their derivatives, with coefficients elements of a given differential field. For such differential forms, it is shown that there exists a theory analogous to a part of van der Waerden's theory (Mathematische Annalen, vol. 96 (1926), p. 183; in particular, pp. 191–192) of ideals of polynomials. (Received July 7, 1932.)

188. Dr. James Singer: *Three-dimensional manifolds and their Heegaard diagrams.*

In this paper the problem of the classification of 3-dimensional manifolds is reduced to the problem of the classification of Heegaard diagrams. Heegaard, in a paper republished in the Bulletin de la Société Mathématique de France, vol. 44 (1916) (see also Veblen, *Analysis Situs*) showed that a 3-dimensional manifold may be represented by a diagram consisting of a closed and connected 2-dimensional manifold upon which are drawn two sets of "canonical" curves. Unfortunately, a manifold may give rise to an infinity of distinct diagrams. We define a set of "moves" which transform a diagram into another, and then say that two diagrams are equivalent if one can be transformed into the other by a finite number of these moves. It is then proved that any two diagrams arising from manifolds equivalent in the sense of semi-linear analysis situs are equivalent, and conversely, equivalent diagrams give rise to manifolds equivalent in the sense of semi-linear analysis situs. (Received June 17, 1932.)

189. Professor Solomon Lefschetz and Dr. J. H. C. Whitehead: *On analytic complexes.*

In his Colloquium Lectures (p. 364) Lefschetz outlined a proof of the theorem according to which an analytic locus can be covered with a simplicial complex. The object of the present paper is to give a detailed proof along the lines there indicated. See also van der Waerden (Mathematische Annalen, vol. 103, pp. 337–362), Koopman-Brown (Transactions of this Society, vol. 34, pp. 231–252). (Received July 8, 1932.)

190. Professor A. A. Albert: *A note on normal division algebras of order sixteen.*

The author recently proved the existence of non-cyclic normal division algebras of order sixteen and showed that his determination of all normal division algebras (Transactions of this Society, vol. 31 (1929), pp. 253–260) gave the best possible result. The determination was long and complicated and a better proof is desirable. The purpose of this short note is to present such a proof. (Received July 14, 1932.)

191. Professor Dunham Jackson: *Orthogonal trigonometric sums.*

This paper extends to the case of trigonometric sums certain properties of systems of polynomials orthogonal over an interval with respect to a given weight function. (Received July 16, 1932.)

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OFFICIAL COMMUNICATIONS

Meetings of the Society have been fixed at the following times and places:

On account of the fact that the Summer Meeting of the Society in 1932 will be held on the Pacific Coast, the meetings that were to have been held at Seattle (June, 1932) and at Los Angeles (November, 1932) will be omitted.

LOS ANGELES, CALIFORNIA, SUMMER MEETING AND COLLOQUIUM, August 30-September 2, 1932.

Abstracts must be in the hands of Associate Secretary M. H. Ingraham, University of Wisconsin, Madison, Wis., not later than July 26. Abstracts received by July 7 will appear in the July issue, or an earlier issue, of this BULLETIN. Professor J. F. Ritt will deliver a series of colloquium lectures on *Differential equations from the algebraic standpoint.* By invitation of the program committee, Professor D. N. Lehmer will speak on *The continued fraction representing cubic and higher irrationalities,* and Professor Tibor Radó will speak on *Recent work in the problem of Plateau.*

NEW YORK CITY, October 29, 1932.

Abstracts must be in the hands of Associate Secretary Tomlinson Fort, 501 West 116th St., New York City, not later than October 8. In order to be printed in the BULLETIN in advance of the meeting, abstracts must be in the hands of the Associate Secretary not later than September 7. By invitation of the program committee, Professor Jesse Douglas will speak on *The problem of Plateau.*

AMES, IOWA, November 25-26, 1932.

Abstracts are due by November 3, 1932.

ATLANTIC CITY, NEW JERSEY, ANNUAL MEETING, December 27-30, 1932.

Abstracts are due by November 30, 1932.

R. G. D. RICHARDSON, *Secretary of the Society.*

Articles for insertion in the BULLETIN should be addressed to E. R. HEDRICK, Editor of the BULLETIN, University of California at Los Angeles. Reviews should be sent to W. R. LONGLEY, Yale University, New Haven, Conn. Notes should be sent to H. W. KUHN, Ohio State University, Columbus, Ohio.

Subscriptions to the BULLETIN, orders for back numbers, and inquiries in regard to non-delivery of current numbers should be addressed to the American Mathematical Society, 450-459 Ahnaip St., Menasha, Wis., or 501 West 116th St., New York.

The initiation fees and the annual dues of members of the Society (see this BULLETIN, p. 322, May, 1930; and the List of Officers and Members, October, 1930, p. 58), are payable to the Treasurer of the Society, Professor G. W. Mullins, 501 West 116th St., New York City.

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